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Application of System Identification to Aircraft at NASA Langley Research Center

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The past, present, and future of system identification applied to aircraft at NASA Langley Research Center (LaRC) in Hampton, Virginia, are discussed. Significant research advances generated at NASA LaRC in the past are summarized, including some perspective on the role these developments played in the practice of system identification applied to aircraft. Selected recent research efforts are described, to give an idea of the type of activities currently being pursued at NASA LaRC. These efforts include real-time parameter estimation, identifying flying qualities models, advanced experiment design and modeling techniques for static wind-tunnel database development, and indicial function identification for unsteady aerodynamic modeling. Projected future developments in the area are outlined.

Introduction

FLIGHT testing and flight test data analysis are inherent activities in aircraft development and the evaluation of aircraft performance, stability, and control. There is a specific interest in obtaining aircraft aerodynamic characteristics from flight data in order to 1) better understand theoretical predictions and wind-tunnel test results and 2) obtain more accurate and comprehensive mathematical models of aircraft aerodynamics, for aircraft simulation, control system design and evaluation, and dynamic analysis.

The purpose of this paper is to present a general approach to aircraft system identification, as formulated at NASA Langley Research Center (LaRC). The paper starts with a historical survey of flight testing and stability and control parameter estimation at NACA Langley Memorial Aeronautical Laboratory, and later at NASA LaRC, leading up to the present state of the art. The individual steps in the methodology are introduced and briefly explained. Four recent example applications of the methodology are given. These examples are mostly related to problems that go beyond traditional stability and control estimation from flight data. The paper is concluded by some thoughts about the future development of the methodology.

The reference list up to and including Ref. 149 is a chronological list of papers and reports generated at NASA LaRC, so the citations in the text are not in order. The references after Ref. 149 were generated elsewhere and are listed in the order cited. These latter references are cited only to clarify the presentation and are in no way representative of the very large body of significant work generated outside LaRC. References 150 and 151, among others, give comprehensive overviews of research in system identification applied to aircraft. The scope of the presentation here is limited to rigid-body aircraft system identification work done at LaRC only.

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History of Flight Testing and Flight Test Data Analysis at Langley Field

In April 1919, the NACA Committee on Aeronautics recommended that some of the work at Langley Field should be devoted to flight testing full-scale machines. This recommendation was realized by a series of flight tests during the summer of 1919. Two advanced airplanes were used in the experiment. The instrumentation for the test was very modest and included an inclinometer, altimeter, tachometer, two airspeed meters, statoscope, and elevator deflection and stick force indicators. The readings of these instruments were taken by an observer at 10-s intervals. The main objective of the tests was to determine the characteristics of the airplane and the extent to which the actual characteristics differed from those predicted from wind-tunnel tests on a model. The flight test results were summarized by Warner and Norton[1] and were presented in graphs, among them angle of attack against indicated airspeed, lift against angle of attack, and elevator deflection and stick force against indicated airspeed.

In the following two years, there were five airplanes at Langley Field available for aerodynamic research. It was realized that future free-flight data should be obtained by recording instruments. For that reason, new sensors were designed and constructed. They were electrically driven and synchronized for taking records on interchangeable film drums. With the new instrumentation system, a series of tests was conducted by Norton and Allen.[2] Data were obtained from steady turning flight and steady sideslips. With the use of the recorded data, the lateral static stability derivatives Y_β, N_β, and L_β were estimated graphically from the slopes of trim curves.

The years 1922 and 1923 brought very rapid development in instrumentation, flight testing, and data analysis. Longitudinal oscillations of a biplane were investigated by Norton and Brown.[3] Norton and Carroll[4] presented measured accelerations along all three body axes of an airplane during different maneuvers. Norton[5] introduced a method for obtaining the damping-in-roll derivative L_p from flight test. The method involved introducing a known rolling moment by sudden release of a known weight from the wing tip then determining the damping graphically from a plot of the roll rate vs time.

In further investigation of longitudinal stability, the period and damping of phugoid oscillations were determined in flight from measured time histories of the airspeed. The same characteristics were obtained by calculation, in which the aerodynamic parameters that entered into the theoretical stability equations were determined in flight by Soulé and Wheatley.[6] A comparison of the period and damping from the two methods indicated that the theory, based on the assumptions of small oscillations, was applicable to the conditions encountered in flight. The flight experiment was repeated in eight different airplanes.

By the late 1930s, a considerable amount of data on control characteristics of airplanes, ranging in size from the smallest training airplane to the largest transports and bombers, was collected by Soulé.[7] The collection of these data formed a basis for investigation of the flying qualities of airplanes.

The 1950s marked the inception of modern system identification methodology applied to aircraft. Two pioneering works by Greenberg[8] at NACA LaRC and Shinbrot[152] at NACA Ames Research Center proposed several techniques for analysis of data from transient maneuvers. These techniques were based on linear and nonlinear least squares methods. Donegan[9] and his colleagues used linear least squares to estimate longitudinal stability and control derivatives and proposed a method for obtaining lateral derivatives from frequency response data (Donegan et al.[11]). Almost simultaneously, Eggleston and Mathews,[10] introduced several methods for estimating frequency response curves and transfer function coefficients.

A major advance in the methodology and its application to aircraft was brought about by the introduction of the digital computer. At NASA LaRC, the first computer program for estimation of stability and control derivatives was developed by Grove et al.[14] in 1972 and gradually was applied to data from various types of aircraft.[12,13,16,17,21,23–25,27,28] Taylor and Iliff[15] also developed a computer program for maximum likelihood output-error parameter estimation that was used at NASA LaRC for many years. An important work containing equations of motion and relationships for correcting various instrumentation errors was written by Gainer and Hoffman[18] in 1972. This report is still used extensively as a reference.

Many theoretical developments took place in the 1970s and early 1980s, in areas such as input design,[19,32] model structure determination,[20,26] real-time parameter estimation,[22,31,61] data compatibility analysis,[29,44] parameter estimation from steady flight measurements,[30] parameter estimation in the frequency domain,[33,38,43] using different parameter estimation methods,[34,37,40,45] and unsteady aerodynamic modeling.[34,39,41]

Further advances in system identification methodology resulted from the arrival of highly augmented, inherently unstable aircraft operating near the stall and in poststall conditions and from two research programs: NASA LaRC General Aviation Stall/Spin Program (1980–1990) and NASA High Angle-of-Attack Technology Program (1988–1998). During the first program, flight data were collected in general aviation aircraft in original and modified configurations, including work in aerodynamic modeling and instrumentation for general aviation aircraft in nonlinear flight regimes associated with spins and stall.[36,50,52,54,56,82] In the second program, the test aircraft was a modified F-18 A fighter, called the F-18 High Alpha Research Vehicle (HARV). The main modifications to this aircraft were the addition of forebody strakes and three-axis thrust vectoring. During roughly the same period, the X-29 and X-31 research aircraft were tested. For the F-18 HARV and X-31 aircraft, a substantial part of the testing was devoted to obtaining data for aerodynamic and thrust-vectoring model parameters at moderate to high angles of attack. Four main problems for continuous development of the methodology appeared:

1) There was a need for data compatibility analysis that would allow removal of systematic instrumentation errors in measured responses, and thus provide more accurate data.

2) An adequate mathematical model for aircraft aerodynamics needed to be determined.

3) Data collinearity existed, caused by high augmentation and/or insufficient excitation of the transient motion.

4) There was a need for a practical input form that could be implemented by a pilot or onboard computer and that would result in higher accuracy of estimated parameters.

Data compatibility analysis developed by Klein and Schiess[29] was based on an extended Kalman filter algorithm. Later, a maximum likelihood method was adopted for the same task.[44] This resulted in a more practical computing technique, documented in Ref. 78. Software based on this approach is in current use at NASA LaRC, and has been very successful in estimating systematic instrumentation errors, resulting in improved data accuracy.

An equation-error algorithm for determining aerodynamic model structure was developed by Klein et al., applying a stepwise regression and used either polynomials,[48,59] or polynomial splines[57,71] in postulated models. Batterson[46,49,55] and Batterson and Klein[62,89] introduced data partitioning to extend the applicability of the approach to large-amplitude maneuvers and highly nonlinear aerodynamics. These methods, which are used extensively at NASA LaRC and elsewhere, have proved to be effective approaches to aerodynamic modeling, particularly for nonlinear aerodynamics and unstable aircraft.

Near-linear dependency (data collinearity) and its effect on equation-error linear regression were discussed in Ref. 83. This was followed by proposed measures for detection and assessment of the collinearity. Two biased estimation methods for parameter estimation were introduced that may reduce the adverse effect of data collinearity. The use of these methods was demonstrated using flight data from the X-29 aircraft.[84,93,99] It was further shown[88,93,99] that reduction in collinearity effects could also be achieved by replacing the pilot stick input by a computer-generated input applied directly to individual control surfaces.

A practical optimal input design technique that generates globally optimal square wave inputs using dynamic programming principles was introduced in Ref. 95 and tested in flight.[100,110,128,134] The technique was also applied to experiments for instrumentation error parameter estimation[101] and closed-loop model identification at high angles of attack.[119] Recently, a different method was developed to optimize multiple inputs for real-time parameter estimation in the frequency domain.[148]

Methodology development efforts also devoted attention to output-error parameter estimation and the accuracy of parameter estimation results. Refinements of the output-error maximum likelihood parameter estimation technique were developed by Murphy and Klein[65] and Murphy[67] and included a new method for calculating confidence intervals for parameters obtained from a nonlinear estimator, such as the output-error maximum likelihood method.[73,79] A different approach to parameter accuracy was developed by Morelli and Klein,[108,124] removing the assumption of white measurement noise. This work allowed calculation of parameter accuracy measures from a single flight test maneuver that are consistent with the scatter in the estimates obtained from repeated maneuvers at the same flight condition addressing the long-standing practical problem of optimistic computed values for estimated parameter error measures. Other significant developments include an optimal Fourier smoothing technique to estimate measurement noise characteristics from measured flight data (see Ref. 117), and the use of the extended Kalman filter for aircraft parameter estimation, with application to the X-31 aircraft (see Refs. 105 and 107).

There has been a great deal of interest in parameter estimation using data transformed from the time domain to the frequency domain. Rigorous treatment of this approach was presented by Klein,[33,43] with examples. Further developments and creation of very capable and widely-used software for aircraft system identification in the frequency domain were done at NASA Ames Research Center[153] and at DLR, German Aerospace Research Center in Germany.[154] For data transformation from the time domain to the frequency domain, an algorithm with high accuracy and flexible frequency selection was developed by Morelli.[125] Later, the frequency-domain formulation was used in estimation of handling qualities models[122,129,140,141,148] and in real-time parameter estimation[131,142] using an equation-error formulation and a recursive Fourier transform. Examples of these applications will be discussed later. Related work was done by Pearson et al.,[111] Skantze et al.,[12] and Pearson[115] using Fourier modulating functions in a formulation that is an extension of the early work by Shinbrot.[152]

System identification methodology at NASA LaRC has been constantly updated and applied to model determination and parameter estimation for a wide variety of aircraft, including general aviation aircraft[47,51,53,58,72]; F-8C fighter aircraft[27,28]; F-14 fighter aircraft[60,66,70,75]; six research fighter aircraft: F-8 with supercritical wing,[24,25] AFTI/F-16,[64,69,77] F-106B,[74] F-18 HARV,[92,96,97,113,120,123,135,140] X-29,[81,84,88,93,99,102] and X-31[105,121]; B-737 transport[68]; Tu-144LL supersonic transport[138,139,141,148]; space shuttle[63]; CH-47 helicopter[42]; a tilt wing vertical/short take-off and landing aircraft[17]; XV-6A aircraft[21,23]; and the DHC-6 Twin Otter commuter aircraft for icing research.[86,87,98] In addition, analysis and modeling was done on data from drop models such as the High Incidence Research Model[76,91,94] and drop models of the X-29[80,90] and X-31.[103–105,107]

In recent years, system identification methodology was also applied to data from static and dynamic wind-tunnel testing. Nonlinear modeling using multivariate orthogonal functions generated from the measured data appeared as a new approach for obtaining global models for aircraft aerodynamics.[114,130,137] This method was also combined with modern experiment design for efficient and accurate development of wind tunnel aerodynamic databases.[143,147] For dynamic wind-tunnel testing, unsteady aerodynamic effects had to be included in the models. For delta-wing aircraft, work by Klein and Noderer[109,116,118] described a formulation using indicial functions for linear models and indicial functionals in formulating nonlinear aerodynamic terms[132] with applications to dynamic wind-tunnel data for the X-31A[133] and the F-16XL.[126,127,144,145] Data were in the form of forced oscillations and/or responses to a ramp change in angle of attack. The methods were also extended to the case of a wing–tail combination.[136]

Over the years, several overviews have been written at NASA LaRC describing system identification applied to aircraft.[40,85,106,127] Recently, algorithms used for system identification applied to aircraft at NASA LaRC have been programmed in MATLAB® and assembled in a toolbox called System IDentification Programs for AirCraft (SIDPAC).[146]

System Identification Applied to Aircraft

When system identification is applied to an aircraft, the equations governing its motion are postulated, and an experiment is designed for obtaining time histories of the input and output variables. The equations of motion are formed by rigid-body force and moment equations in body axes[155]

$$m\dot{\boldsymbol{V}} + \boldsymbol{\omega} \times m\boldsymbol{V} = \boldsymbol{F}_G + \boldsymbol{F}_T + \boldsymbol{F}(\boldsymbol{V}, \boldsymbol{\omega}, \boldsymbol{u}, \theta)$$

$$\boldsymbol{I}\dot{\boldsymbol{\omega}} + \boldsymbol{\omega} \times \boldsymbol{I}\boldsymbol{\omega} = \boldsymbol{G}(\boldsymbol{V}, \boldsymbol{\omega}, \boldsymbol{u}, \theta) \tag{1}$$

and by a set of kinematic equations relating the Euler attitude angles and body-axis angular velocities. In Eq. (1), m is the mass, $\boldsymbol{I}$ is the inertia matrix, $\boldsymbol{V}$ and $\boldsymbol{\omega}$ are the linear and angular velocity vectors, and $\boldsymbol{u}$ is the control vector. The vectors $\boldsymbol{F}_G$ and $\boldsymbol{F}_T$ represent the gravity and thrust forces, $\boldsymbol{F}$ and $\boldsymbol{G}$ represent aerodynamic force and moment, respectively, and $\boldsymbol{\theta}$ is a vector of parameters that specify aerodynamic characteristics of the aircraft.

For system identification, the aircraft equations of motion are augmented by output and measurement equations. The complete set of equations can be written as

$$\dot{\boldsymbol{x}} = \boldsymbol{f}[\boldsymbol{x}(t), \boldsymbol{u}(t), \boldsymbol{\theta}], \qquad \boldsymbol{x}(0) = \boldsymbol{x}_0, \qquad \boldsymbol{y} = \boldsymbol{h}[\boldsymbol{x}(t), \boldsymbol{u}(t), \boldsymbol{\theta}]$$

$$z(i) = \boldsymbol{y}(i) + \boldsymbol{v}(i), \qquad i = 1, 2, \ldots, N \tag{2}$$

where the state vector $\boldsymbol{x}$ is comprised of $\boldsymbol{V}$, $\boldsymbol{\omega}$, and Euler angles and $\boldsymbol{u}$ is a vector of controls. The output vector $\boldsymbol{y}$ contains the aircraft responses. The measured outputs $z(i)$ at each sampling time t_i are corrupted by measurement noise $\boldsymbol{v}(i)$, and the number of data points is N.

Aircraft identification can be defined as follows: Aircraft identification is a determination, from input and output measurements, of a structure for $\boldsymbol{F}$ $(\boldsymbol{x}, \boldsymbol{u}, \boldsymbol{\theta})$ and $\boldsymbol{G}(\boldsymbol{x}, \boldsymbol{u}, \boldsymbol{\theta})$ and estimation of unknown parameters $\boldsymbol{\theta}$ in $\boldsymbol{F}(\boldsymbol{x}, \boldsymbol{u}, \boldsymbol{\theta})$ and $\boldsymbol{G}(\boldsymbol{x}, \boldsymbol{u}, \boldsymbol{\theta})$.

In many practical applications, the structure of $\boldsymbol{F}(\boldsymbol{x}, \boldsymbol{u}, \boldsymbol{\theta})$ and $\boldsymbol{G}(\boldsymbol{x}, \boldsymbol{u}, \boldsymbol{\theta})$ is assumed to be known, and aircraft identification is reduced to parameter estimation. A general approach to aircraft identification adopted at NASA LaRC is shown in Fig. 1 in the form of a block diagram. Various steps in the procedure include model postulation, design of an experiment, data compatibility analysis, model structure determination and parameter estimation combined with collinearity diagnostics, and model validation.

Model Postulation

Model postulation is influenced by the type of maneuver used for system identification and by prior knowledge about aircraft aerodynamics. The aerodynamic forces and moments are expressed in the form of polynomials or polynomial splines as

$$C_a = C_{a_0} + \sum_{j=1}^{n-1} \theta_j x_j \tag{3}$$

where C_a is the aerodynamic coefficient, C_{a_0} is the value of the coefficient at a steady-state initial condition, and x_j now represent input variables and aircraft motion variables, their polynomial combinations (multivariable Taylor series terms), and/or spline terms. The postulated model is then used in model structure determination and parameter estimation.

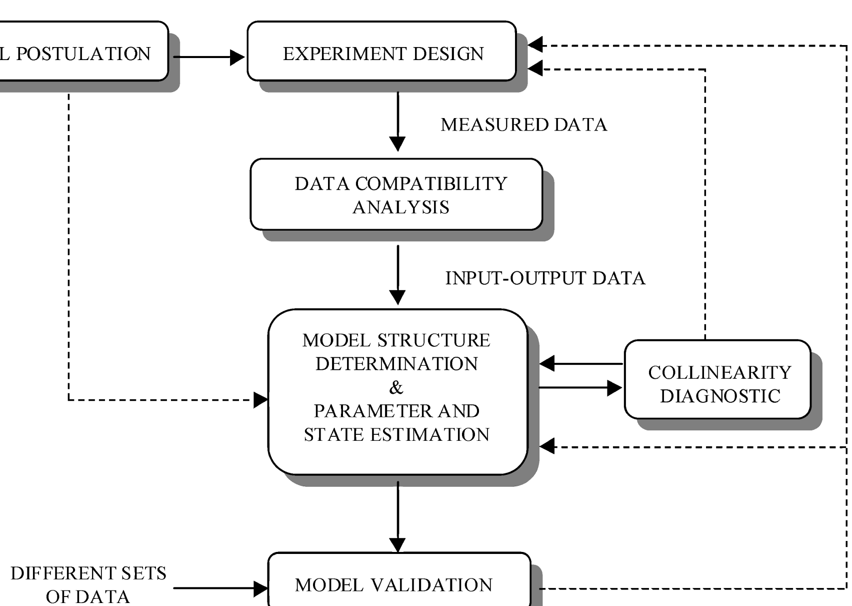


Fig. 1 System identification applied to aircraft.

For a model with unsteady aerodynamics, the model for the force and moment coefficients can be formulated in terms of indicial functions,[109,156]

$$\boldsymbol{C}_a(t) = C_{a_0} + \int_0^t C_{a\xi}[t-\tau, \xi(\tau)]^T \frac{\mathrm{d}}{\mathrm{d}\tau}\xi(\tau)\,\mathrm{d}\tau \qquad (4)$$

where $\boldsymbol{\xi}$ is a vector of aircraft state and input variables on which the coefficient C_a depends, and $\boldsymbol{C}_{a\xi}$ is a vector of indicial functions whose elements are the responses in C_a to unit steps in ξ. The indicial responses $\boldsymbol{C}_{a\xi}$ are functions of elapsed time $(t-\tau)$ and are continuous single-valued functions of $\boldsymbol{\xi}(t)$. The indicial functions approach steady-state values with increasing values of the argument $(t-\tau)$. If the indicial response $\boldsymbol{C}_{a\xi}$ is only a function of elapsed time, Eq. (4) is simplified as

$$C_a(t) = C_{a_0} + \int_0^t \boldsymbol{C}_{a\xi}(t-\tau)^T \frac{\mathrm{d}}{\mathrm{d}\tau}\boldsymbol{\xi}(\tau)\,\mathrm{d}\tau \qquad (5)$$

When analytical forms of indicial functions are specified, the aerodynamic model based on Eq. (4) or (5) can be used in the aircraft equations of motion for stability and control studies that involve either nonlinear or linear unsteady aerodynamics, respectively. The resulting equations of motion will be represented by a set of integro–differential equations.

Experiment Design

An important part of the experiment design is the selection of input forms. It has been recognized that the shape of an input signal can influence the accuracy of estimated parameters from flight measurements. Attempts at obtaining parameter estimates with high accuracy led many researchers to the development of an optimal input. One of the latest techniques for optimal input design is discussed in Refs. 95 and 128.

Data Compatibility Analysis

In practice, the measured response data, even after careful handling, can still contain bias and scale factor errors due to the characteristics of the sensors. To verify data accuracy, a compatibility check can be applied to the measured aircraft responses. This check includes aircraft state estimation, based on known kinematics and the available sensor measurements, estimation of unknown instrumentation errors, and a comparison of reconstructed responses with those measured. The state equations are formed by kinematic relationships, and the parameter vector usually contains constant biases and scale factor errors. The estimation techniques are similar to those used in estimation of states and aerodynamic parameters; see Refs. 29 and 78.

State and Parameter Estimation

The problem of state estimation is usually reduced to integration of aircraft equations of motion, provided that these equations represent a deterministic system, that is, no process noise and no random parameters in the equations. The objective of the parameter estimation is to find values of unknown parameters $\boldsymbol{\theta}$ from noisy measurements z and known input $\boldsymbol{u}$. An estimator $\hat{\boldsymbol{\theta}}(z)$ is then defined as a function of random variable z. Parameter estimation requires specification of 1) model structure and parameters to be determined, 2) measurement vector z, and 3) mathematical models for z and uncertainties in parameters $\boldsymbol{\theta}$ and measurement vector z.

Three estimation techniques can be developed, depending on the properties of the measurement equation. If this equation is linear in the parameters, then it can be formulated as

$$z = \boldsymbol{H}\boldsymbol{\theta} + \boldsymbol{v} \qquad (6)$$

The measurement equation that is nonlinear in the parameters has the form

$$z = \boldsymbol{h}(\boldsymbol{\theta}) + \boldsymbol{v} \qquad (7)$$

In both equations, the matrix $\boldsymbol{H}$ and the form of the vector $\boldsymbol{h}(\boldsymbol{\theta})$ are assumed to be known, and the measurement vector z contains n_z elements. Furthermore, the vector of measurement errors is considered at the ith measurement, where $i = 1, 2, \ldots, N$, and N is the number of data points.

As to the uncertainties in the parameters and the measurements, three models can be considered. They are designated according to Schweppe[157] as the Bayesian model, the Fisher model, and the Least Squares model, formed as follows:

Bayesian Model:

i) $\boldsymbol{\theta}$ is a vector of random variables specified by its probability density $p(\theta)$.

ii) $\boldsymbol{v}$ is a random vector specified by its probability density $p(\boldsymbol{v})$.

Fisher Model:

i) $\boldsymbol{\theta}$ is a vector of unknown parameters.

ii) $\boldsymbol{v}$ is a random vector specified by its probability density $p(\boldsymbol{v})$.

Least Squares Model:

i) $\boldsymbol{\theta}$ is a vector of unknown parameters.

ii) $\boldsymbol{v}$ is a random vector of measurement noise.

For measurement equations that are nonlinear in the parameters, Eq. (6) in all three models is replaced by Eq. (7).

Estimator for Bayesian Model

The development of an estimator in the Bayesian model follows from the Bayesian estimation theory explained in Refs. 157 and 158. This estimator has not been established in aircraft parameter estimation because of the unavailability of the probability density $p(\boldsymbol{\theta})$. However, a Bayes-like estimator can be formulated as a biased estimation technique in linear regression (see Ref. 84).

Estimator for Fisher Model

An estimator for the Fisher model is based on the Fisher information theory[159] using the concept of a likelihood function:

$$L(z;\boldsymbol{\theta}) \equiv p(z|\boldsymbol{\theta})$$

Because now $\boldsymbol{\theta}$ is not a random variable, the density function $p(\boldsymbol{\theta})$ is not defined. The most common estimator for the Fisher model is the maximum likelihood (ML) estimator, which is equal to the value of $\boldsymbol{\theta}$ that maximizes $L(z;\boldsymbol{\theta})$ for given z. In the case of Gaussian distribution of $p(z)$, the likelihood function takes the form

$$L(z;\boldsymbol{\theta}) = [(2\pi)^{n_z}|\boldsymbol{R}|]^{-\frac{1}{2}} \exp\left\{-\tfrac{1}{2}[z-\boldsymbol{H}\boldsymbol{\theta}]^T\boldsymbol{R}^{-1}[z-\boldsymbol{H}\boldsymbol{\theta}]\right\} \qquad (8)$$

Rather than maximize the likelihood function, it is more convenient to minimize its negative logarithm. The ML estimator for parameters of a dynamic system with no process noise and a sequence of measurements $z(i)$ can be expressed as

$$\hat{\boldsymbol{\theta}} = \min_{\theta} \sum_{i=1}^{N} [-\ln L(z(i);\boldsymbol{\theta})] \qquad (9)$$

Then the ML estimator is reduced to an output-error method with the cost functional

$$J = \frac{1}{2}\sum_{i=1}^{N} \boldsymbol{v}(i)^T\boldsymbol{R}^{-1}\boldsymbol{v}(i) + \frac{N}{2}\ln|\boldsymbol{R}| \qquad (10)$$

where $\boldsymbol{v}(i) = z(i) - \boldsymbol{y}(i, \boldsymbol{\theta}_0)$ are the residuals and $\boldsymbol{\theta}_0$ is a vector of nominal values for the parameters. Experience shows that a suitable technique for minimization of the cost functional in Eq. (10) is the modified Newton–Raphson method (see Ref. 15). When this technique is used, the step size $\Delta\hat{\boldsymbol{\theta}}$ for the parameter estimation is given by

$$\Delta\hat{\boldsymbol{\theta}} = -\boldsymbol{M}^{-1}\frac{\partial J}{\partial\boldsymbol{\theta}} \qquad (11)$$

where $\boldsymbol{M}$ is the Fisher information matrix

$$\boldsymbol{M} = \sum_{i=1}^{N}\left[\frac{\partial\boldsymbol{y}(i)}{\partial\boldsymbol{\theta}}\right]^T\boldsymbol{R}^{-1}\left[\frac{\partial\boldsymbol{y}(i)}{\partial\boldsymbol{\theta}}\right] \qquad (12)$$

and the cost gradient $\partial J/\partial \boldsymbol{\theta}$ is

$$\frac{\partial J}{\partial \boldsymbol{\theta}} = \sum_{i=1}^{N} \left[\frac{\partial \boldsymbol{y}(i)}{\partial \boldsymbol{\theta}}\right]^T \boldsymbol{R}^{-1}\boldsymbol{v}(i) \tag{13}$$

The noise covariance matrix $\boldsymbol{R}$ is estimated using a relaxation procedure with the equation resulting from setting the gradient $\partial J/\partial \boldsymbol{R}$ equal to zero and solving for $\boldsymbol{R}$,

$$\hat{\boldsymbol{R}} = \frac{1}{N}\sum_{i=1}^{N} \boldsymbol{v}(i)\boldsymbol{v}(i)^T \tag{14}$$

The information matrix also provides a lower bound on the parameter covariances, that is,

$$\operatorname{cov}(\hat{\boldsymbol{\theta}}) = E\{(\hat{\boldsymbol{\theta}} - \boldsymbol{\theta})(\hat{\boldsymbol{\theta}} - \boldsymbol{\theta})^T\} \geq \boldsymbol{M}^{-1} \tag{15}$$

Expression (15) is valid if the measurement noise is random, Gaussian, and white. Numerous analyses from measured flight data have shown, however, that the residuals can be far from white. The residuals very often contain some deterministic components. The result is colored residuals, leading to a new expression for the parameter covariance matrix[124] in the form

$$\operatorname{cov}(\hat{\boldsymbol{\theta}}) = \boldsymbol{M}^{-1}\left[\sum_{i=1}^{N}\left(\frac{\partial \boldsymbol{y}(i)}{\partial \boldsymbol{\theta}}\right)^T \boldsymbol{R}^{-1}\sum_{j=1}^{N} \boldsymbol{\Re}_{vv}(i-j)\boldsymbol{R}^{-1}\left(\frac{\partial \boldsymbol{y}(j)}{\partial \boldsymbol{\theta}}\right)\right]\boldsymbol{M}^{-1} \tag{16}$$

where $\Re_{vv}(i-j)$ is the autocorrelation matrix for the output residual vector $\boldsymbol{v}$. The autocorrelation matrix is estimated from

$$\boldsymbol{\Re}_{vv}(k) = \frac{1}{N-k}\sum_{i=1}^{N-k} \boldsymbol{v}(i)\boldsymbol{v}(i+k)^T = \boldsymbol{\Re}_{vv}(-k) \tag{17}$$

Further simplification of the ML method is obtained by assuming that both the input and state variables are measured without errors. This assumption leads to an equation-error method that can be formulated as a linear regression to model the aerodynamic dependencies. Reasons for using ML methods can be summarized as follows:

1) The method in its general form can estimate parameters of a linear or nonlinear dynamic system that is either deterministic or stochastic and where the measurement noise is random but not necessarily Gaussian.

2) The method assumes known inputs and noisy measured aircraft motion variables, which is consistent with typical flight-test measurements.

3) Parameter estimates are asymptotically consistent, unbiased, and efficient. However, these properties hold strictly only when there is no deterministic modeling error, which is seldom achieved in practice.

4) The technique can be generalized to the frequency domain, in which case the data from more than one maneuver can be easily combined for a single analysis, and the approach can be applied to unstable aircraft flying with stability augmentation systems. The last two items can also be accomplished using ML in the time domain, but with difficulty.

Estimator for Least Squares Model

When the form of the least squares model is specified, no model for uncertainty in $\boldsymbol{\theta}$ and $\boldsymbol{v}$ is used; i.e., there is no probability statement about $\boldsymbol{\theta}$ and $\boldsymbol{v}$. An estimator for the least squares model can be obtained by the reasoning that, given $\boldsymbol{z}$, the best estimator of $\boldsymbol{\theta}$ follows from minimizing

$$J = (\boldsymbol{z} - \boldsymbol{H\theta})^T\boldsymbol{R}^{-1}(\boldsymbol{z} - \boldsymbol{H\theta}) \tag{18}$$

where $\boldsymbol{R}^{-1}$ is now a positive definite weighting matrix. In the special case where $\boldsymbol{R}^{-1} = \boldsymbol{I}$, the ordinary least squares estimate follows from the cost function

$$J = \sum_{i=1}^{N}[\boldsymbol{z}(i) - \boldsymbol{H}(i)\boldsymbol{\theta}]^2 \tag{19}$$

when the entire set of measured data $z(i)$, $i = 1, 2, \ldots, N$, is considered.

When the states and inputs in the aerodynamic model equations are replaced by their measured values, the regression equations are formed as

$$y(i) = \theta_0 + \theta_1 x_1(i) + \cdots + \theta_p x_p(i) + \varepsilon(i), \qquad i = 1, 2, \ldots, N \tag{20}$$

where y represents a nondimensional aerodynamic coefficient, x_1 to x_p the regressors, and ε is the error. The N equations can be also be expressed as

$$\boldsymbol{Y} = \boldsymbol{X\theta} + \boldsymbol{\varepsilon} \tag{21}$$

where $\boldsymbol{Y} = [y(1), y(2) \ldots, y(N)]^T$, etc., and the notation has changed from before ($\boldsymbol{Y} = \boldsymbol{z}$, $\boldsymbol{X} = \boldsymbol{H}$, and $\boldsymbol{\varepsilon} = \boldsymbol{v}$) to correspond to conventional notation for regression problems. The least squares (LS) parameter estimates are obtained as

$$\hat{\boldsymbol{\theta}} = (\boldsymbol{X}^T\boldsymbol{X})^{-1}\boldsymbol{X}^T\boldsymbol{Y} \tag{22}$$

and their covariance matrix as

$$\operatorname{cov}(\hat{\boldsymbol{\theta}}) = \sigma^2(\boldsymbol{X}^T\boldsymbol{X})^{-1} \tag{23}$$

For white noise, the error variance σ^2 can be estimated from

$$\hat{\sigma}^2 = \frac{1}{N}\sum_{i=1}^{N}[y(i) - \hat{y}(i)]^2 \tag{24}$$

or independently of the parameter estimates by use of smoothing techniques.[117] For colored noise,

$$\operatorname{cov}(\hat{\boldsymbol{\theta}}) = (\boldsymbol{X}^T\boldsymbol{X})^{-1}\left[\sum_{i=1}^{N}\boldsymbol{x}(i)\sum_{j=1}^{N}\Re_{ee}(i-j)\boldsymbol{x}(j)^T\right](\boldsymbol{X}^T\boldsymbol{X})^{-1} \tag{25}$$

where $\Re_{ee}(i-j)$ is the autocorrelation function of residuals

$$e(i) = y(i) - \hat{y}(i) = y(i) - \boldsymbol{x}(i)^T\hat{\boldsymbol{\theta}} \tag{26}$$

and $\boldsymbol{x}(i)$ is the ith row of the regressor matrix $\boldsymbol{X}$.

For regressors that are deterministic quantities, with white noise on the measured output, the LS parameter estimates are unbiased, efficient, and consistent. In real applications, however, the regressors are measured with errors. Then the LS estimates are biased, inefficient, and inconsistent. Despite these properties, the linear regression is a widely used estimation method for the following reasons:

1) It is a simple, noniterative method for parameter estimation.

2) The LS estimates serve as nominal starting values for the ML methods.

3) Linear regression can be applied to data generated by partitioning an ensemble of data from repeated measurements, with respect to one or more variables.[89]

4) Data from more than one maneuver can be easily combined for a single analysis.

5) Linear regression can be extended to a technique for model structure determination, e.g., stepwise regression.

6) Formulation of a regression problem can be used for investigation of near-linear dependence (collinearity) among measured state and input variables, and for the development of biased estimation techniques for dealing with highly collinear data.

7) The technique can be applied to unstable aircraft flying with stability augmentation systems.

8) The technique generalizes easily to frequency-domain data.

Model Structure Determination

A major problem in system identification is the selection, based on measured data, of an adequate (parsimonious) model. An adequate model has a structure sufficient to characterize the data, facilitates the successful estimation of unknown parameters whose existence can be substantiated, and has good prediction capabilities. For model structure determination, stepwise regression methods[48,57] can be used. In this approach, the determination of a model for the aerodynamic coefficients includes three steps: postulation of terms that might enter the model, selection of an adequate model, and validation of the model selected. After the aerodynamic model equations are posulated, significant terms among the candidates are determined based on partial correlation coefficients and F ratios, and the corresponding parameters are estimated. At every step of the stepwise regression, the terms incorporated into the model in previous stages and a new term entering the model are reexamined for their significance.

The model structure can be determined in a methodical fashion when the candidate regressors are orthogonalized.[114,147] Using a predicted squared error criterion, which combines a measure of the candidate model fit to the measured data with a model overfit penalty term, an adequate model with good predictive capability can be identified.

Collinearity Diagnostic

The augmentation of high-performance aircraft very often introduces near-linear relationships among the input and output variables (data collinearity). When linear regression is used in data analysis, the collinearity results in an ill-conditioned $\boldsymbol{X}^T\boldsymbol{X}$ matrix in expression (22) for LS parameter estimates. Because of that, the collinearity can cause computational problems and reduce the accuracy of parameter estimates. Procedures for detection of collinearity are recommended in Ref. 83 and are applied to flight data. These procedures are as follows:

1) Examine the correlation matrix $\boldsymbol{X}^{*T}\boldsymbol{X}^*$ and its inverse, where the matrix $\boldsymbol{X}^*$ is formed by centered and scaled regressors.

2) Eigenvalue analysis of the $\boldsymbol{X}^T\boldsymbol{X}$ matrix or singular value decomposition of the $\boldsymbol{X}$ matrix.

3) Decompose the parameter variance into a sum of components, each corresponding to one of the eigenvalues of the $\boldsymbol{X}^T\boldsymbol{X}$ matrix or singular values of the $\boldsymbol{X}$ matrix.

If the collinearity reveals a serious problem, some way of dealing with it should be chosen. Additional data can be used, the experiment can be redesigned, the model can be respecified, or different techniques from the ordinary LS procedure can be used. One of these techniques, the mixed estimation, is introduced in Ref. 83. Mixed estimation (ME) is a procedure that uses prior information on parameters to augment measured data, instead of a probability density. It is closely related to a Bayesian estimator of $\boldsymbol{\theta}$. Mixed estimation includes the usual regression model given by Eq. (21) and the additional assumption that a set of prior conditions on $\boldsymbol{\theta}$ can be written as

$$\boldsymbol{d} = \boldsymbol{A}\boldsymbol{\theta} + \boldsymbol{\zeta} \tag{27}$$

In this equation, $\boldsymbol{A}$ is a matrix of known constants and $\boldsymbol{\zeta}$ is a vector of random variables with $E\{\boldsymbol{\zeta}\} = \boldsymbol{0}$ and $E\{\boldsymbol{\zeta}\boldsymbol{\zeta}^T\} = \sigma^2\boldsymbol{W}$, where $\boldsymbol{W}$ is a known weighting matrix. When Eqs. (21) and (27) are combined, the mixed model is obtained. For known σ^2, the application of LS to this model results in the mixed estimator,

$$\hat{\boldsymbol{\theta}}_{\mathrm{ME}} = (\boldsymbol{X}^T\boldsymbol{X} + \boldsymbol{A}^T\boldsymbol{W}^{-1}\boldsymbol{A})^{-1}(\boldsymbol{X}^T\boldsymbol{Y} + \boldsymbol{A}^T\boldsymbol{W}^{-1}\boldsymbol{d}) \tag{28}$$

It is shown in Ref. 84 that the addition of prior information to the ordinary regression results in reduction of parameter variance. In real applications of the ME, the values of $\boldsymbol{A}$, $\boldsymbol{W}$, and $\boldsymbol{\sigma}$ are not known exactly; therefore, the resulting estimator is biased.

Model Validation

Model validation is the last step in the identification process and should be applied regardless of the complexity of the estimation method. The identified model must demonstrate that its parameters have physically reasonable values with acceptable accuracy and that it has good prediction capabilities. For these reasons, the parameter estimates are compared with any information available about the aircraft aerodynamics, which can come from theoretical predictions, computational fluid dynamics, wind-tunnel measurements, or results of other flight tests using different maneuvers and/or different estimation techniques. During these comparisons, the accuracy of the parameter estimates must be taken into account. Prediction capabilities of the identified model are checked on a set of data not used in the identification process.

Recent Applications of System Identification to Aircraft

System identification techniques have been applied to a variety of aircraft and experiments at NASA LaRC. The following material is a sampling of some recent applications. The descriptions are intentionally brief, due to space limitations, so that the interested reader should consult the references for full details. The applications described here are: real-time parameter estimation on a Twin Otter commuter aircraft, nonlinear aerodynamic modeling of wind-tunnel data for a flying model aircraft, low-order equivalent system modeling on the F-18 HARV fighter aircraft and the Tu-144LL supersonic transport aircraft, and unsteady aerodynamic modeling of wind-tunnel forced oscillation data for an F-16XL fighter model.

Real-Time Parameter Estimation

Estimating model parameters and associated error bounds in real time can provide significant advantages for efficient flight testing, flight envelope expansion, and real-time monitoring for flight safety.[131] In addition, indirect reconfigurable control methods require real-time estimates of stability and control derivatives. At NASA LaRC, a frequency-domain equation-error technique was developed for estimating time-varying stability and control derivatives in real time.[142] The method uses a recursive Fourier transform at selectable frequencies in an equation-error formulation. Some advantages of this approach are automatic removal of noise, measurement biases, and infrequent data dropouts; enhanced signal-to-noise ratio; low computational and memory requirements; and accurate parameter error bounds without corrections.

Researchers from NASA LaRC and NASA John H. Glenn Research Center collaborated to demonstrate this real-time parameter estimation method in flight on the NASA Twin Otter Icing Research Aircraft, as part of the NASA Smart Icing Systems program. Figure 2 shows a piloted elevator doublet sequence and the measured pitch rate response at a low-angle-of-attack flight condition. Figures 2d and 2e show the time histories of two of the dimensional stability and control derivatives estimated using real-time parameter estimation in the frequency domain. The state-space model for the short-period motion was

$$\begin{bmatrix} \dot{\alpha} \\ \dot{q} \end{bmatrix} = \begin{bmatrix} Z_\alpha & Z'_q \\ M_\alpha & M_q \end{bmatrix} \begin{bmatrix} \alpha \\ q \end{bmatrix} + \begin{bmatrix} Z_{\delta_e} \\ M_{\delta_e} \end{bmatrix} \delta_e \tag{29}$$

where the Z'_q parameter includes the inertial term, i.e., $Z'_q = 1 + Z_q$. The dotted lines in Figs. 2d and 2e indicate the postflight estimate of each derivative, using time-domain output-error parameter estimation.[146] The real-time frequency-domain algorithm needed very little information to converge to the postflight values, and the error bounds accurately reflect the quality of the parameter estimates throughout the maneuver. This real-time analysis was done with a 450-MHz laptop computer carried aboard the Twin Otter aircraft. The data were sent from the onboard flight data recording system to the laptop computer over a serial data line at 20 Hz. The laptop computer was able to import the data and carry out the required calculations in real time, with enough time and computational power left over to generate plots similar to Fig. 2.

Recently, research has been done to develop a multiple-input design technique for real-time frequency-domain parameter estimation.[149] Inputs were designed to be mutually orthogonal in the time domain and the frequency domain and to have very low

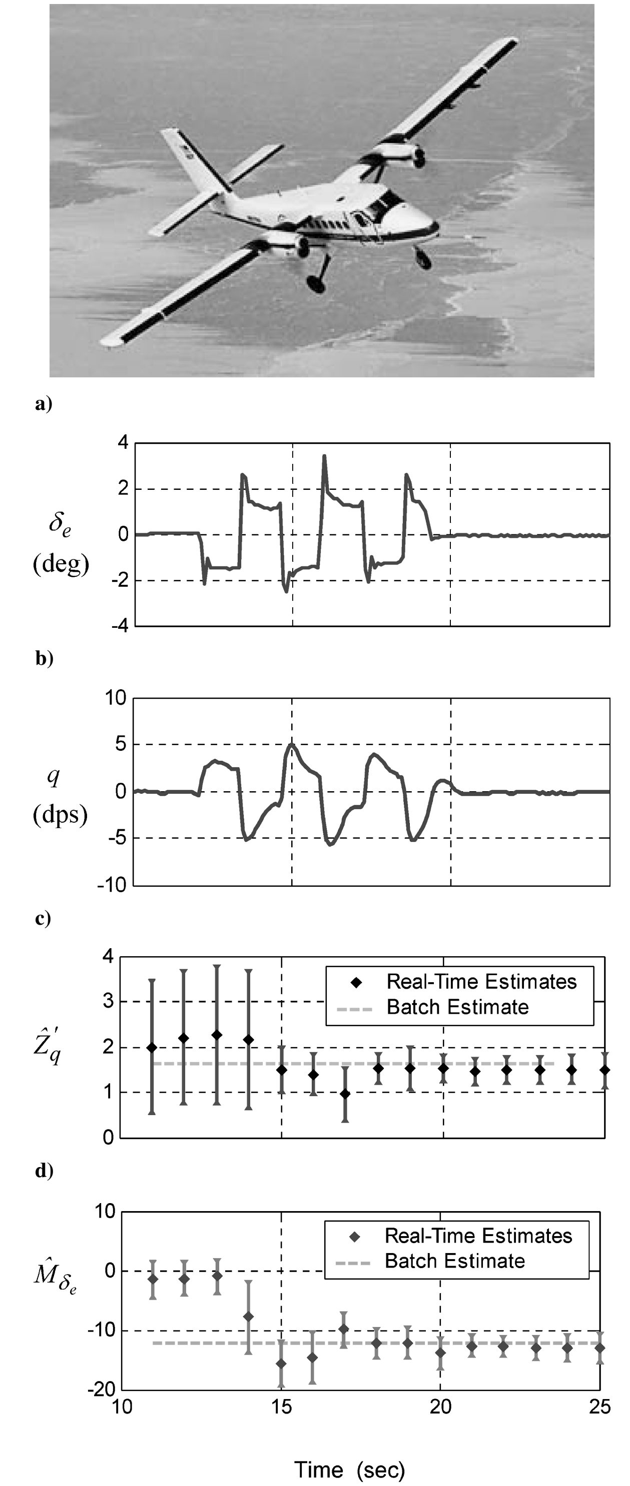


Fig. 2 Twin Otter real-time parameter estimation, $\alpha_0 = 0.6$ deg, and $M_0 = 0.21$.

peak factor, so that the aircraft can be excited sufficiently for accurate parameter estimation without large aircraft response amplitudes. This would allow continuous, accurate real-time estimation of time-varying stability and control derivatives with minimal disturbances to the aircraft and pilot.

Wind-Tunnel Database Development

For modern aircraft with multiple control effectors and large flight envelopes, it is important that efficient wind-tunnel testing methods be used to collect aerodynamic data. Improved insight is obtained if the aerodynamic dependencies can be characterized by multivariate polynomial models, rather than extensive tables of numbers. Multivariate polynomial models for the aerodynamic force and moment coefficients also enable important analysis and design work, such as bifurcation analysis, full-envelope control system design, and simulation database updates using flight-test data. At NASA LaRC, development of a multivariate orthogonal polynomial modeling technique[114] provided the capability to identify global models for aerodynamic data from wind-tunnel tests.[143] This modeling technique was combined with modern experiment design to create an effective and efficient procedure for characterizing aircraft aerodynamics. Reference 147, describes how this was done using a flying model aircraft called the Free-flying Aircraft for Subscale Experimental Research (FASER), which is being developed at NASA Langley for fundamental aircraft dynamics and control research. The wind-tunnel testing for FASER covered a large flight envelope and included propulsion-induced effects at low angles of attack and sideslip angles. Figure 3 shows an identified multivariate polynomial model surface fit to rolling moment coefficient data, for a three-dimensional slice through a subspace of the independent variable space for the full flight envelope of the vehicle. The × markers represent measured data points, and the smooth surface is the identified model. Because the model surface can only be shown for a three-dimensional slice through a higher-order (five-dimensional) space of independent variables, only a few of the 80 total data points for this subspace appear in Fig. 3, as shown by the × markers. The remaining data points lie in other parts of the five-dimensional independent variable subspace. A single multivariate polynomial model was identified to fit all of the data in the subspace, which means that similar plots could be made for other parts of the independent variable subspace with the same single identified model. With use of the multivariate orthogonal function technique, the multivariate modeling of the aerodynamic dependencies, including both the nonlinear multivariate model structure determination and subsequent parameter estimation, was done in an automated fashion, based only on the data and statistical modeling metrics, without the need for analyst judgement.

Combining these analytical methods for experiment design and multivariate polynomial modeling with automated wind-tunnel rig control and real-time data acquisition has the potential to provide large improvements in testing efficiency, model accuracy, and real-time awareness of the data quality and modeling results.

Flying Qualities Model Identification

Low-order models characterizing the aircraft response to pilot inputs are useful for quantifying flying qualities and evaluating control system performance. These models, called low-order equivalent system (LOES) models, have a fixed structure corresponding to classical aircraft dynamics, with an input time delay to account for time lag due to high-frequency closed-loop control, digitization, and various nonlinearities. Recent work at NASA LaRC has resulted in development of a frequency-domain approach for identifying LOES models from flight-test data, particularly for shorter maneuvers excited by doublets and multistep inputs. The approach is based on the work by Klein[33] and employs the advanced frequency-domain transformation developed in Ref. 125, with optimization methods tailored for application to the LOES modeling problem.[140] High-accuracy Fourier transform data at selectable frequencies are used for the modeling, in contrast to the traditional approach of using spectral estimates to generate a Bode plot. Because of this, the new approach makes it possible to obtain accurate LOES modeling results using relatively short data records.

The approach has been applied to high-angle-of-attack tracking data for the F-18 HARV aircraft[140] and also to the Tu-144LL supersonic transport aircraft.[148] Figure 4 shows data for three identical longitudinal tracking tasks flown at approximately 40-deg angle of attack by three different test pilots on the F-18 HARV. Pilots A, B, and C rated the aircraft flying qualities at level 1, 2, and 3, respectively, even though the task, aircraft, and flight-condition were very nearly the same in each case. Analysis of the flight-test data using the approach described earlier showed that each pilot's flying qualities rating was completely consistent with the LOES model identified from the measured data. Table 1 contains parameter estimates for the longitudinal LOES model of pitch rate $\tilde{q}$ to longitudinal stick input $\tilde{\eta}_e$ in the frequency domain,

$$\frac{\tilde{q}}{\tilde{\eta}_e} = \frac{(b_1 s + b_0)}{s^2 + a_1 s + a_0} e^{-\tau s} = \frac{K_\theta (s + 1/T_{\theta_2})}{s^2 + 2\,\zeta_{sp}\omega_{sp} s + \omega_{sp}^2} e^{-\tau s} \tag{30}$$

At the bottom of Table 1, the flying qualities levels for each pilot's rating are shown, along with flying qualities levels predicted using the identified LOES and information in the relevant military specification.

The difference in the pilots' flying qualities ratings was not from any wide variation in their rating technique, but rather was the result of different aircraft responses to the way each pilot flew the tracking task, as indicated by the power spectrum of the pilot inputs shown in the bottom of Figs. 4b–4d. (Note the change in vertical scale for pilot C, Fig. 4d.) The predominantly low frequency inputs of pilot A resulted in a level 1 rating, whereas pilot B used a more even frequency spectrum and rated the airplane level 2. Pilot C used high-amplitude inputs at low and midrange frequencies, which resulted in control surface rate limiting, and experienced a level 3 closed-loop airplane response.

The LOES modeling approach provided the capability to identify accurately analytical LOES models using data from a relatively short tracking task that was not intended to provide data for LOES modeling. Accurate identified LOES models were necessary to illuminate the situation.

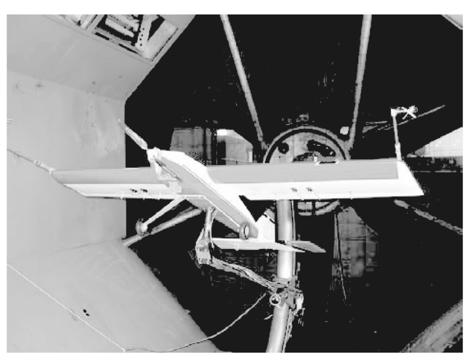
Figure 5 shows that the LOES modeling approach described above worked well for the Tu-144LL at a transonic flight condition using a multistep maneuver. The LOES model identified from the data shown in Fig. 5b was used to predict the response shown in Fig. 5c. Figure 5c demonstrates that the identified model has good prediction capability for a maneuver at the same flight condition using an input with inverted polarity. The data from the prediction maneuver was not used in the model identification.

Table 1 Longitudinal LOES modeling results for F-18 HARV longitudinal tracking data, $\alpha_0 = 40$ deg

	Estimate (standard error)		
Parameter	Maneuver 376d	Maneuver 321e	Maneuver 320j
b_1	0.787 (0.103)	0.745 (0.152)	0.638 (0.041)
b_0	1.401 (0.493)	3.346 (0.701)	0.451 (0.203)
a_1	3.785 (0.678)	4.371 (0.675)	4.670 (0.314)
a_0	15.49 (3.39)	33.84 (4.50)	10.30 (1.53)
τ	0.026 (0.018)	0.042 (0.020)	0.236 (0.018)
$1/T_{\theta_2}$	1.78	4.49	0.71
ω_{sp}	3.94	5.82	3.21
ζ_{sp}	0.48	0.38	0.73
Pilot flying qualities (FQ) level/LOES model FQ level	1/1	2/2	3/3

Unsteady Aerodynamic Modeling Using Indicial Functions

For better understanding of aircraft aerodynamics in rapid, large-amplitude maneuvers, NASA LaRC conducted a series of wind-tunnel tests on models of the F-16XL aircraft. These tests included measurements of aerodynamic forces and moments under static



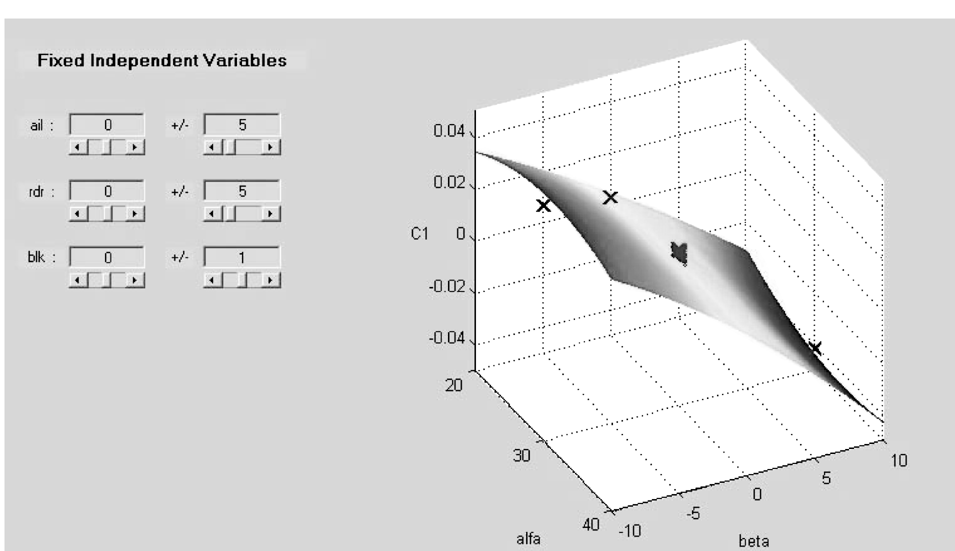
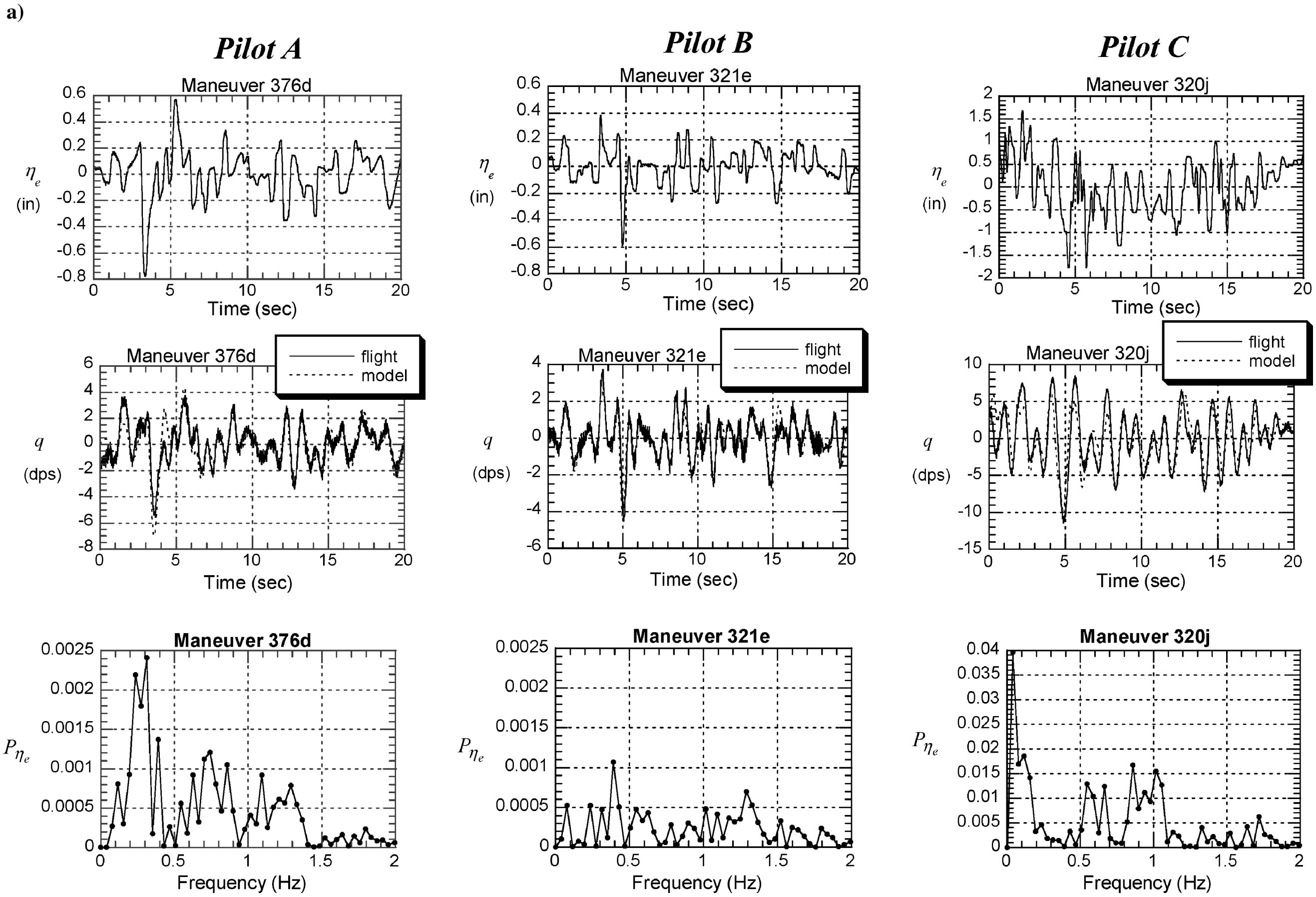


Fig. 3 FASER wind-tunnel database development, C_l subspace 20, $20 \leq \alpha \leq 40$ deg, and $-10 \leq \beta \leq 10$ deg.

a)



b) c) d)

Fig. 4 F-18 HARV LOES modeling, tracking task, $\alpha_0 \approx 40$ deg, and $M_0 \approx 0.32$.

conditions, followed by oscillatory and ramp tests. In the following example, model formulation and identification for an aircraft performing a large-amplitude one-degree-of-freedom motion in pitch is considered. Detailed development is given in Ref. 127.

A general model for the lift coefficient can be expressed in terms of indicial functions as

$$C_L(t) = C_L(0) + \int_0^t C_{L_\alpha}[t-\tau;\alpha(\tau), q(\tau)]\dot{\alpha}(\tau)\,\mathrm{d}\tau + \frac{\ell}{V}\int_0^t C_{L_q}[t-\tau;\alpha(\tau), q(\tau)]\dot{q}(\tau)\,\mathrm{d}\tau \tag{31}$$

where $C_L(0)$ is the value of the lift coefficient at initial steady-state conditions, α and q are the angle of attack and pitch rate, respectively, t represents time, and τ is the time delay. Assuming that the indicial function reaches steady state as $t \to \infty$, and neglecting the effect of $\dot{q}$, the model for $C_L(t)$ can be simplified as

$$C_L(t) = C_L(\alpha) + \frac{\ell}{V}C_{L_q}(\alpha)q(t) - \int_0^t F_{L_\alpha}[t-\tau;\alpha(\tau)]\dot{\alpha}(\tau)\,\mathrm{d}\tau \tag{32}$$

with

$$F_{L_\alpha}(t;\alpha) = e^{-b(\alpha)t}a(\alpha) \tag{33}$$

In Eqs. (32) and (33), ℓ is the characteristic length and C_{L_q} is the damping-in-pitch derivative; $a(\alpha)$ and $b(\alpha)$ are polynomials in α.

In the resulting model, the term $C_L(\alpha)$ can be determined from static wind-tunnel tests. The remaining terms $C_{L_q}(\alpha)$,

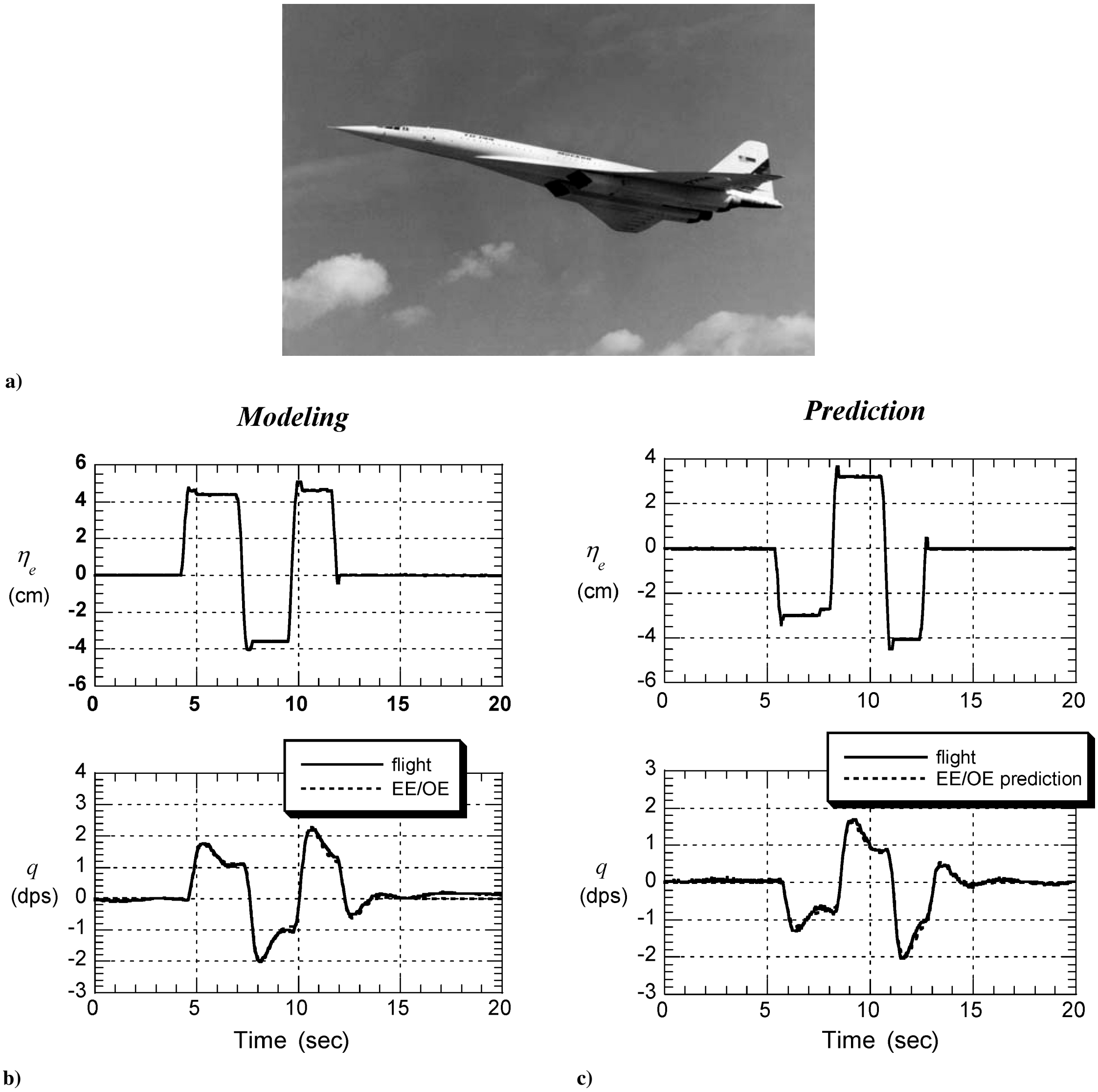


Fig. 5 Tu-144LL LOES modeling, multistep maneuvers, $\alpha_0 = 5.8$ deg, and $M_0 = 0.88$.

$a(\alpha)$, and $b(\alpha)$ can be identified from measured oscillatory data $C_L(\alpha : \alpha_0, \alpha_A, k)$, where α_0 is a nominal value of α, α_A is the amplitude of the oscillation, and $k = \omega\ell/V$ is the reduced frequency, with angular velocity ω.

The identification of the three unknown polynomials utilizes a two-stage procedure that combines stepwise regression and the maximum likelihood estimator. The resulting model was validated by its prediction ability. In Figs. 6a and 6b, a comparison of measured C_L and C_L predicted by the identified model is shown for oscillatory and ramp data, respectively. None of these data were used in the modeling, so Figs. 6a and 6b are both prediction cases using a model identified from other data. Figure 6a also shows measured lift coefficient under static conditions. From the results shown, it can be concluded that the identified model explains the variation in C_L for the two selected dynamic maneuvers very well.

Future Development of System Identification for Aircraft Applications

The future of system identification applied to aircraft has many interesting possibilities. Interaction between system identification practitioners and computational fluid dynamics (CFD) researchers should increase substantially over time. In addition to helping to validate CFD results, aircraft system identification might be used in cooperative approaches with CFD, to take advantage of the strengths of both approaches or having one approach fill in gaps where the other cannot be used effectively. Currently, identified aerodynamic models are lumped parameter models, which characterize the aerodynamics of the entire aircraft as a whole. With the introduction of small and inexpensive sensors, and increasing computational capabilities, it may be advantageous to identify distributed parameter models for aircraft aerodynamics. The result would be that flowfields for individual parts of the aircraft could be taken into account, which could be useful for describing motions such as a spin. However, the distributed models will have many more parameters than traditional lumped parameter models. To realize improved fidelity, the distributed parameter models will generally require more data, which will in turn require new methods for experiment design, data handling, and modeling. Along these same lines, modern air vehicles are tending toward having more control effectors, with a wider variation in their type and operation. This will require efficient wind-tunnel testing and modeling methods, as well as efficient flight-test experiment design and modeling. In the long term, the aim could be toward automating the complete experimentation, data analysis, and modeling process, first in the wind tunnel, then possibly in flight. The role of the system identification engineer would then evolve toward high-level oversight of the automated processes and trouble-shooting specific problems that arise.

Many modern flight vehicles, particularly vehicles of smaller sizes and vehicles that may use smart materials or morphing structures, will have significant structural dynamic interactions with the aerodynamics. This will require additional system

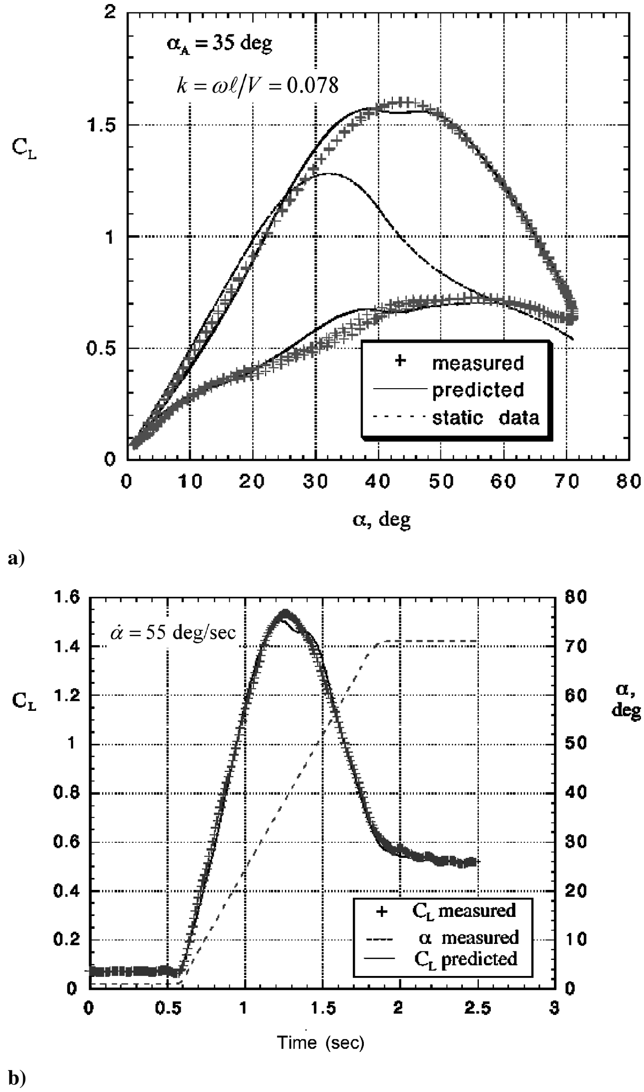


Fig. 6 F-16XL measured and predicted lift coefficient in a) oscillatory test and b) ramp test.

identification work, including appropriate analytical formulations of the models, with parameters that can be accurately identified from measured data.

There is still plenty of work to be done in the area of nonlinear and unsteady aerodynamic modeling, including understanding the fundamental phenomena, designing appropriate experiments, building ground facilities or aircraft flight-testing capabilities that will allow the necessary experimentation to be conducted in a safe and cost-effective manner, formulating the appropriate model forms, estimating the model parameters accurately, and developing appropriate tests and criteria for validating the identified models. The forefront for this work is in the areas of static and dynamic wind-tunnel testing. The phenomena being studied are complex, so extensive experimentation and iterative modeling efforts will be required. This highlights the need for cost-effective means for doing the requisite experiments.

More work will be required to create a practical real-time parameter estimation system that can work consistently without being noticed by the pilot. In particular, more work is needed on data information content required for successful real-time parameter estimation and on evaluation of the effectiveness of using time-varying stability and control derivatives throughout the flight envelope to characterize the aerodynamics. Payoffs for this research are very high because real-time parameter estimation can be important for efficient flight testing, flight envelope expansion, reconfigurable control, and real-time stability and control monitoring for improved flight safety.

Another area for future research is the rapid generation of aerodynamic databases from flight-test data and the related topic of efficiently updating existing simulation aerodynamic databases, based on flight test data. These areas have important implications for rapid development of both experimental and production aircraft.

Undoubtedly, more applications and theoretical problems in system identification will arise that cannot be foreseen. Every new vehicle or device must be tested at some point. The fundamental activity called system identification, which involves identifying mathematical constructs that fit measured data from an experiment for purposes of understanding, control, and prediction, will continue to be an essential activity in aerospace engineering.

Summary

A historical overview of research done at NASA Langley Research Center in the area of system identification applied to rigid-body aircraft has been presented. The material included here was not intended as a comprehensive overview of all work in the field, but rather as a historical look at the efforts made at one particular research institution, NASA Langley Research Center, and its predecessor NACA Langley. Researchers at this institution have been involved in applying system identification to aircraft since shortly after the invention of the airplane. After the review of significant work up to the present time, four recent research projects at NASA Langley demonstrating applications of system identification methodology to aircraft modeling problems are described in some detail. Finally, speculations on promising directions for future work in the field are made.

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